Overview of FRC-related modeling (July 2014–present)

Artan Qerushi

AFRL-UCLA Basic Research Collaboration Workshop

January 20th, 2015

AFTC PA Release# 15009, 16 Jan 2015

Distribution A—Approved for public release; distribution unlimited.
Outline

1. FRC configuration
   - Illustration of FRC configuration
   - FRC formation with RMF

2. 2D \((r - \theta)\) model of RMF-formed FRCs
   - Original publications of RMF-formed FRCs
   - Model equations and their numerical solution

3. Collision Radiative model
   - 0D model equations
   - Test calculations

4. Magnetized plasma closure for electron-ion-neutral mixture
   - Magnetized plasma closure for electron-ion-neutral mixture
   - Applied field modules

5. Conclusions

Distribution A–Approved for public release; distribution unlimited.
Illustration of FRC configuration

- $\theta$–pinch formed FRC.

**Figure 2. Schematic of an FRC confined in single turn coil.**
FRC formation with RMF (Rotating Magnetic Field)

- RMF-formed FRC.
Original publications of modeling RMF-formed FRCs

Illustration of 2D \((r - \theta)\) RMF model

**Figure 1.** Schematic diagram showing the application of a transverse rotating field to a magnetized plasma column.
2D \((r - \theta)\) model equations and their numerical solution

- Coupled equations for \(B_z(r, \theta)\) and \(A_z(r, \theta)\)

\[
\frac{\partial A_z}{\partial t} = \frac{\eta}{\mu_0} \nabla^2 A_z + \frac{1}{ne e\mu_0 r} \left[ \left( \frac{\partial A_z}{\partial r} \right) \left( \frac{\partial B_z}{\partial \theta} \right) - \left( \frac{\partial A_z}{\partial \theta} \right) \left( \frac{\partial B_z}{\partial r} \right) \right]
\]

\[
\frac{\partial B_z}{\partial t} = \frac{\eta}{\mu_0} \nabla^2 B_z + \frac{1}{ne e\mu_0 r} \left[ \left( \frac{\partial A_z}{\partial \theta} \right) \frac{\partial}{\partial r} \nabla^2 A_z - \left( \frac{\partial A_z}{\partial r} \right) \frac{\partial}{\partial \theta} \nabla^2 A_z \right]
\]

- Solve numerically by expanding \(B_z\) and \(A_z\) in Fourier series and deriving 1D equations for the Fourier coefficients (coupled).
- Advance the equations for the Fourier coefficients in time using an ODE solver (method of lines).
0D model equations

\[
\frac{d n_e}{d t} = \sum_{k=1,Z} q_i n_i
\]

\[
\frac{d n_k}{d t} = n_e (n_{k-1} S_{k-1} - n_k (S_k + \alpha_k) + n_{k+1} \alpha_{k+1}), \quad k = 1, 2, \ldots, Z - 1
\]

\[
\frac{d n_Z}{d t} = n_e (n_{Z-1} S_{Z-1} - n_Z \alpha_Z),
\]

\[
\frac{d T_e}{d t} = \sum_{k=0,Z} n_k L_k(T_e)
\]

- All coefficients \(S_k, \alpha_k, L_k\) are functions of \(T_e\).

- NIST (FLYCHK code) provides these coefficients for all the periodic table in the temperature range 0.5 [ev] to 100 [kev].

- Have implemented in C++ the coefficients for H, He, O, Ar, Kr and Xe. Little work needed to add other elements if that is needed.
Test calculation for Oxygen


Distribution A–Approved for public release; distribution unlimited.
Test calculation for Oxygen (time-dependent radiation loss)


Distribution A–Approved for public release; distribution unlimited.
Magnetized plasma closure (Introduction)

\[
\frac{\partial (\rho_j \vec{v}_j)}{\partial t} = \cdots - \nabla \cdot \vec{\Pi}_j + \sum_k \vec{R}_{jk}
\]

\[
\frac{\partial \epsilon_e}{\partial t} = \cdots - \nabla \cdot \vec{q}_j - \vec{\Pi}_j : \nabla \vec{v}_j + \sum_k Q_{jk}
\]

- \cdots indicate terms not related to collisions,
- \(\vec{R}_{jk}\) represents transfer of momentum between the species \(j\) and \(k\) due to collisions,
- \(\vec{q}_j\) represents flux of heat due to the temperature gradient \(\nabla T_j\) and Ohmic heating,
- \(\vec{\Pi}\) represents the off-diagonal part of the pressure tensor,
- \(Q_{jk}\) represents collisional energy transfer between the species \(j\) and \(k\).

This term is proportional to the temperature difference \(T_k - T_j\).

The purpose of a fluid closure is to provide explicit expressions for \(\vec{R}_{jk}, \vec{q}_j, \vec{\Pi}\) and \(Q_{jk}\) in terms of \(n_j, \vec{v}_j, T_j\) and the magnetic field strength.
Magnetized plasma closure (literature)

Magnetized plasma closure (results)


Distribution A–Approved for public release; distribution unlimited.
Conclusions

- We have implemented in stand-alone C++ code
  - The CR data for H, He, O, Ar, Kr and Xe.
  - Magnetized plasma closure for electron-ion-neutral mixture.
  - Applied field modules for the FRC experiment setup including the DC coils and the RMF antenna.

- The 2D $r - \theta$ model is currently implemented in modern fortran. A C++ version which uses the SUNDIALS ODE solver suite is being written to be included in our software framework (SMURF).

- Work on multi-dimensional fluid models using the Finite Element method and unstructured grids is ongoing and will be reported in the near future.

Distribution A–Approved for public release; distribution unlimited.
Extra Slides.
Assumptions of 2D RMF model

- Frequency condition for current drive
  \[ \omega_{ci} < \omega < \omega_{ce}. \]

- Infinitely long plasma cylinder lying in a uniform magnetic field. All quantities are assumed independent of axial location \( z \).

- The ions form a uniformly distributed neutralizing background of fixed, massive positive charges.

- The plasma resistivity, \( \eta \), is taken to be a scalar quantity which is constant in time and uniform on space. In particular, \( \eta \) is assumed to be of the form
  \[ \eta = \frac{m_e \nu_{ei}}{n_e e^2} \]

- Electron inertia is neglected; that is it is assumed that \( \omega \ll \omega_{ce}, \nu_{ei} \).

- The displacement current is neglected. This implies that only systems for which \( \omega r_p / c \), where \( r_p \) is the plasma radius, are considered.
FRC formation with RMF (Rotating Magnetic Field)

- Illustration of 2D \((r - \theta)\) RMF formation FRC.
Test calculation for Iron


Distribution A–Approved for public release; distribution unlimited.